

Polinomios

1) Indicar si es o no es un polinomio:

(*) $2 + x^3 - 5x^5 + 6x^{10}$ (Si)

(*) $\sqrt{x} + 2\sqrt[3]{x} + 5\sqrt{x} + 1$ (No)

(*) $1 + x + x^2 + x^3 + \dots$ (No)

(*) $1 + x^2 - \frac{1}{x^2} + x^3 + \frac{1}{x^3}$ (No)

(*) $\sqrt{2}x^5 + \sqrt{3}xy^2 + \sqrt{5}x^3y^3$ (Si)

(*) $\frac{1}{2}xy^2 + \frac{1}{3}x^4y^5 - \frac{1}{5}x^7y^3$ (Si)

2) $P(x, y) = 5x^2 - 2xy^3 - 3y^4$
Hallar $P(3; -2)$

Solución

Datos
$$\begin{aligned} x=3 & \left\{ \begin{aligned} P(3; -2) &= 5(3)^2 - 2(3)(-2)^3 - 3(-2)^4 \\ y=-2 & \left\{ \begin{aligned} P(3; -2) &= 5 \cdot (9) - 2 \cdot (3) \cdot (-8) - 3 \cdot (+16) \end{aligned} \right. \end{aligned} \right. \end{aligned}$$

$$P(3; -2) = 45 + 48 - 48$$

$$P(3; -2) = 45$$

3) $H(x) = 3x^2 - x + 1$

$$Q(x) = x^2 + x + 1$$

Hallar: $M = [H(-2) - Q(3)]^{H(1)}$

Solución

(*) Dato: $H(-2)$

$$x = -2 \left\{ \begin{aligned} H(-2) &= 3(-2)^2 - (-2) + 1 \\ H(-2) &= 3(4) + 2 + 1 \end{aligned} \right.$$

$$H(-2) = 12 + 3$$

$$H(-2) = 15$$

(*) Dato: $Q(3)$

$$x = 3 \left\{ \begin{aligned} Q(3) &= (3)^2 + (3) + 1 \\ Q(3) &= 9 + 3 + 1 \end{aligned} \right.$$

$$Q(3) = 13$$

(*) Dato: $H(1)$

$$x = 1 \left\{ \begin{aligned} H(1) &= 3(1)^2 - (1) + 1 \\ H(1) &= 3(1) - 1 + 1 \end{aligned} \right.$$

$$H(1) = 3$$

reemplazando en "M":

$$M = [H(-2) - Q(3)]^{H(1)}$$

$$M = [15 - 13]^3$$

$$M = (2)^3$$

$$M = 8$$

4) $P(x) = x^2 - x + 1$

Calcular: $P(P(P(2)))$

Solución

Dato $P(2)$

$$x = 2 \left\{ \begin{aligned} P(2) &= (2)^2 - (2) + 1 \\ P(2) &= 4 - 2 + 1 \end{aligned} \right.$$

$$P(2) = 3$$

$P(2) = 3$; reemplazando en:

$$\Rightarrow P(P(3))$$

Dato $P(3)$

$$x = 3 \left\{ \begin{aligned} P(3) &= (3)^2 - (3) + 1 \\ P(3) &= 9 - 3 + 1 \end{aligned} \right.$$

$$P(3) = 7$$

$P(3) = 7$; reemplazando en:

$$\Rightarrow P(7)$$

Dato $P(7)$

$$x = 7 \left\{ \begin{aligned} P(7) &= (7)^2 - (7) + 1 \\ P(7) &= 49 - 7 + 1 = 43 \end{aligned} \right.$$

5) $P(x) = 2x + 3$

Calcular:

$P(a+1) - P(a-1)$

Solución

Dato $P(a+1)$

$x = a+1$ $\left\{ \begin{array}{l} P(a+1) = 2(a+1) + 3 \\ P(a+1) = 2a + 2 + 3 \\ P(a+1) = 2a + 5 \end{array} \right.$

Dato: $P(a-1)$

$x = a-1$ $\left\{ \begin{array}{l} P(a-1) = 2(a-1) + 3 \\ P(a-1) = 2a - 2 + 3 \\ P(a-1) = 2a + 1 \end{array} \right.$

reemplazando:

$\therefore P(a+1) - P(a-1)$
 $\downarrow \quad \quad \downarrow$
 $(2a+5) - (2a+1)$
 $2a+5 - 2a-1$
 4

6) $P(2x-1) = x^2 - 2$

Calcular:

$K = \sqrt{\frac{P(7) + P(5) - 3}{2}}$

Solución

Dato $P(3)$
 $\left\{ \begin{array}{l} 2x-1=3 \\ 2x=4 \\ x=2 \end{array} \right. \left\{ \begin{array}{l} P(3) = (2)^2 - 2 \\ P(3) = 4 - 2 \\ P(3) = 2 \end{array} \right.$

7) Dato: $P(7)$

$\left\{ \begin{array}{l} 2x-1=7 \\ 2x=8 \\ x=4 \end{array} \right. \left\{ \begin{array}{l} P(7) = (4)^2 - 2 \\ P(7) = 16 - 2 \\ P(7) = 14 \end{array} \right.$

Dato: $P(5)$

$\left\{ \begin{array}{l} 2x-1=5 \\ 2x=6 \\ x=3 \end{array} \right. \left\{ \begin{array}{l} P(5) = (3)^2 - 2 \\ P(5) = 9 - 2 \\ P(5) = 7 \end{array} \right.$

reemplazando:

$K = \sqrt{\frac{P(7) + P(5) - 3}{2}}$

$K = \sqrt{\frac{14 + 7 - 3}{2}}$

$K = \sqrt{\frac{18}{2}}$

$K = \sqrt{9}$

$K = 3$

7) $P(x-3) = x + 2$

Calcular $P(x)$

Solución

Dato $P(x)$

$\left\{ \begin{array}{l} x-3=x \\ x=x+3 \end{array} \right. \left\{ \begin{array}{l} P(x) = x+3+2 \\ P(x) = x+5 \end{array} \right.$

8) $P(3x-1) = 5x+1$
 Calcular $P(P(5)+6)$
Solución

Dato : $P(5)$

$$\left. \begin{array}{l} 3x-1=5 \\ 3x=6 \\ x=2 \end{array} \right\} \begin{array}{l} P(5) = 5(2)+1 \\ P(5) = 10+1 \\ P(5) = 11 \end{array}$$

reemplazamos en:

$$\begin{array}{l} P(P(5)+6) \\ \downarrow \\ P(11+6) \\ P(17) \end{array}$$

*Dato ↓

$$\left. \begin{array}{l} 3x-1=17 \\ 3x=18 \\ x=6 \end{array} \right\} \begin{array}{l} P(7) = 5(6)+1 \\ P(7) = 30+1 \\ P(7) = 31 \end{array}$$

9) $P(x) = 3x+2$

$$P(Q(x)) = 6x+5$$

Hallar $Q(2)$

Solución

Dato

$$\left. \begin{array}{l} x=Q(x) \\ x=Q(x) \end{array} \right\} \begin{array}{l} P(Q(x)) = 3(Q(x)) + 2 \\ 6x+5 = 3 \cdot Q(x) + 2 \\ 6x+5-2 = 3Q(x) \\ 6x+3 = 3Q(x) \\ \frac{6x+3}{3} = Q(x) \end{array}$$

$$\frac{6x}{3} + \frac{3}{3} = Q(x)$$

$$2x+1 = Q(x)$$

∴ Dato : $Q(2)$

$$\left[x=2 \right] \left\{ \begin{array}{l} Q(2) = 2(2)+1 \\ Q(2) = 4+1 \\ Q(2) = 5 \end{array} \right.$$

10)

$$P\left(1-\frac{1}{x}\right) = 4x^2 - 2x - 5$$

Calcular: $P(3)$

Solución

Dato : $P(3)$

$$1-\frac{1}{x} = 3$$

$$1-3 = \frac{1}{x}$$

$$-2 = \frac{1}{x}$$

$$-2x = 1$$

$$x = \frac{1}{-2}$$

$$x = -\frac{1}{2}$$

$$P(3) = 4\left(-\frac{1}{2}\right)^2 - 2\left(-\frac{1}{2}\right) - 5$$

$$P(3) = 4\left(\frac{1}{4}\right) + \frac{2}{2} - 5$$

$$P(3) = 1 + 1 - 5$$

$$P(3) = 2 - 5$$

$$P(3) = -3$$

11) $P(x) = 5x+3$. Calcular: $P(P(x))$

Solución

Dato: $P(P(x))$

$$\left. \begin{array}{l} x=P(x) \\ x=P(x) \end{array} \right\} \begin{array}{l} P(P(x)) = 5(P(x)) + 3 \end{array}$$

$$P(P(x)) = 5(5x+3) + 3$$

$$P(P(x)) = 25x + 15 + 3$$

$$P(P(x)) = 25x + 18$$